

EMITTANCE MEASUREMENT: QUADRUPOLE SCAN

INTRODUCTION:

The quadrupole scan technique is but one of a number of (linear) beam optics methods used for time-integrated emittance measurements. It is based on the fact that the squared rms beam radius is proportional to the quadrupole “strength”, or inverse focal-length f squared, so

$$\langle x^2 \rangle = A \left(\frac{1}{f^2} \right) - 2AB \left(\frac{1}{f} \right) + (C + AB^2), \quad (1a)$$

$$= m_1 l_{quad}^2 - 2m_1 m_2 l_{quad} + (m_3 + m_1 m_2^2), \quad (1b)$$

where A, B, C, m_1, m_2, m_3 are constants, l_q is the quadrupole current, and

$$\frac{1}{f} = \kappa l_{eff} = \frac{qg_{he} l_{he}}{\gamma m_e \beta c} l_q. \quad (2)$$

The effective length (hard-edge model) of the quadrupole is $l_{he} = 5.164$ cm, and the effective on-axis quadrupole gradient (per amp) is $g_{he} = 2.60$ G/cm A. (Note that the actual peak gradient is different, see Tech Note UMER-060220-SB for details if you're interested – you don't need it for the analysis).

The emittance and Twiss parameters are calculated according to

$$\varepsilon = \frac{\sqrt{AC}}{d^2}, \quad (3)$$

$$\alpha = \sqrt{\frac{A}{C}} \left(B + \frac{1}{d} \right), \beta = \sqrt{\frac{A}{C}}, \gamma = \frac{1 + \alpha^2}{\beta}. \quad (4)$$

The results above apply to a beam with negligible space charge, although they can be applied as a first order calculation for space-charge dominated beams as well [see Anderson, *et al*, PRST-AB, 5, 014201 (2002)]. In UMER, we can produce an emittance-dominated beam, the so-called “pencil beam” with a 0.25 mm (radius) aperture. The distance from the aperture to the phosphor screen in IC2 is 79.6cm. The beam current is $600 \mu A \pm 5\%$ at 10 keV. The distance from the quadrupole (Q2 -the one whose strength we vary in this experiment) to the phosphor screen diagnostics used for beam size measurements is $d = 26.2$ cm.

BACKGROUND: Minty and Zimmermann, “Beam Techniques-Beam Control and Manipulation”, USPAS Notes, June 14-25, 1999. Also, Sections 3.8.2 and 3.5 in M. Reiser’s *Theory and Design of Charged Particle Beams*, and UMER Technical Note UMER-060220-SB, “New” Approach to Effective Length and Strength of UMER Quadrupoles.

EQUIPMENT:

UMER matching section, CCD camera, and image-processing software.

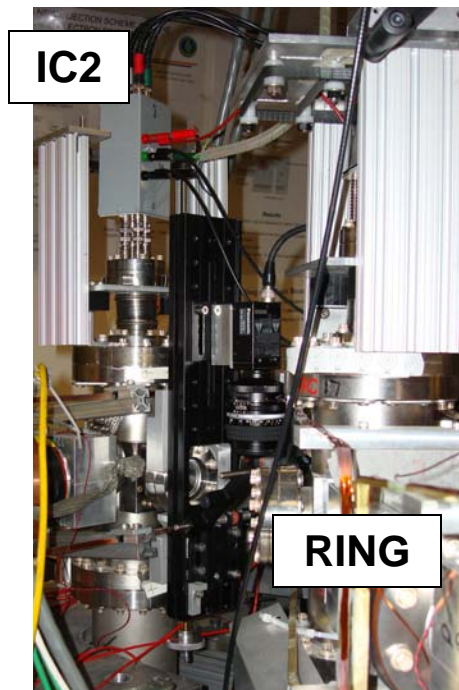
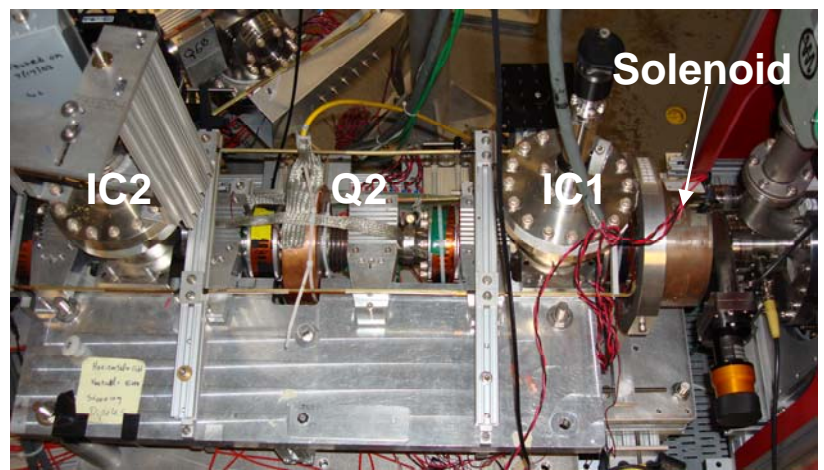


Figure 1: Top view of UMER matching section (bottom), and side view of intersection of matching section and ring (left). The camera is setup vertically on IC2.



PROCEDURE:

1. Set the aperture at the pinhole marking (290 deg.).
2. Set up the CCD camera at IC2, and position the fluorescent screen in place with the actuator (the gate valve must be closed during this latter operation).
3. Turn on the Helmholtz coils over the matching section.
4. Power the solenoid with a current of 4.92 A, and leave all quadrupoles off for now. Also load the standard steering solution¹.
5. Open the gate valve and turn the beam on at 10 kV. Measure the beam current with the Bergoz transformer and record the value.

¹ A slight variation may work better: SD1H=0, SD1V=0.25A, SD2H=-1.15A, SD2V=0.35A, SD3H=0.079A, SD3V=0.

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6. Take a test beam picture with zero current applied to quadrupole Q2. Make sure that the camera is focused. If the sensor is saturated, close the lens iris diaphragm as needed.
7. Also check for camera software settings for minimum background noise.
8. Take beam pictures for currents applied to Q2 from 0.0 A to 2.0 A in steps of 0.25 A. Take 2-3 extra pictures for Q2 currents that yield beam sizes near the beam waist (minimum beam size). Save the files in a format that you can use for processing latter.
9. Repeat the previous step with opposite quadrupole polarity (the polarity switch is over the Lambda Power Supply rack).
10. Time permitting, repeat the experiment with the 7 mA beam (aperture setting = 50 deg.)

ANALYSIS / QUESTIONS:

1. Record the electron gun operating conditions and tabulate all your measurements including errors.
2. Derive the relationships between the constants A , B , and C in Eq. 1a and the constants m_1 , m_2 , and m_3 in Eq. 1b.
3. Use your favorite image processing software (MatLab or MATHCAD scripts may be provided, or you can use J-Image) to determine the 2RMS beam dimensions in X (horizontal) and Y (vertical) planes for each quadrupole current. It's important that you use the right mm/pixel calibration and that background subtraction be implemented if the pictures require it. Tabulate your results.
4. Plot the squared beam X -dimension vs. quadrupole current. Do the same for the squared Y -dimension.
5. Do a least-square fit of your data from the previous step. Follow the form of Eq. 1b.
6. Determine the horizontal and vertical emittances, according to Eq. 3.
7. Determine the Twiss parameters (Eq. 4).
8. Calculate the focal lengths of your quadrupole (see Eq. 2) for each current and compare with the quadrupole's effective length (5.164 cm). Based on your results, comment on the validity of the thin-lens approximation for the beam-optics calculations that lead to Eqs. 1a-b, etc.
9. (Bonus) If you did the experiment with the 7 mA beam as well, can the quadrupole scan technique be valid in this case? Comment. [see Anderson, *et al*, PRST-AB, 5, 014201 (2002)].

NOTE: You may find that some of the data points, especially those for large quad currents may "throw" the quality of the data fit (this may have relevance to question 8 above). Try the fit with a reduced data set to see if your fit improves. A Chi-squared (a measure of the quality of the least-squares fit) of less than about 10 or 20 is fine. Notice that m_3 cannot be negative, or else you get an imaginary emittance.